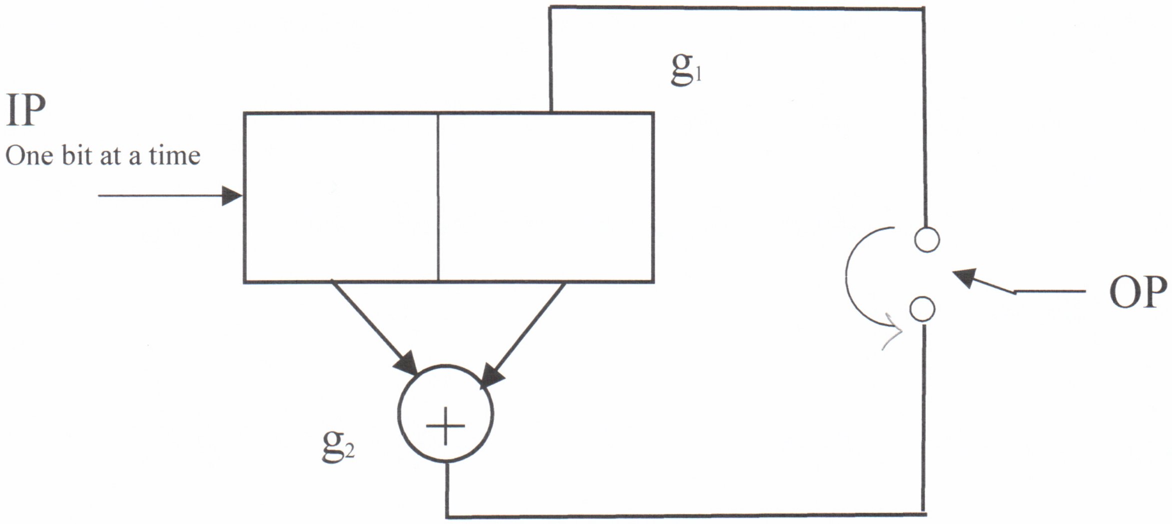
Çankaya University – ECE Department – ECE 587

Student Name :
Student Number :

Duration : 2 hours
Open book exam

Questions

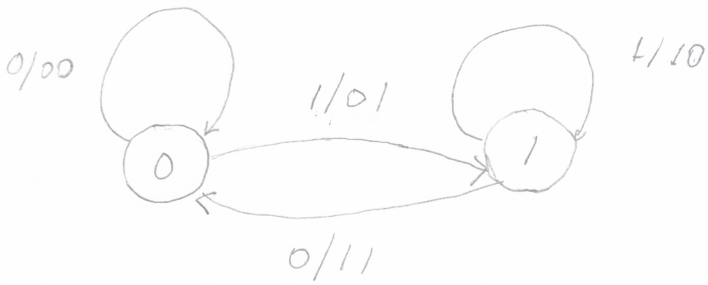
1. (35 Points) For the convolution encoder shown below, determine the code rate (k / n), constrain length (L), find the generator sequences ($\mathbf{g}_1$, $\mathbf{g}_2$), draw the state and trellis diagrams. Assuming 110101 is fed to this encoder, find the output of the encoder, and at receiver side, feed this output to the trellis diagram and perform the decoding. Comment on your results



Solutions 1: $k = 1$, $n = 2$, code rate $k/n = 0.5$, $L = 2$

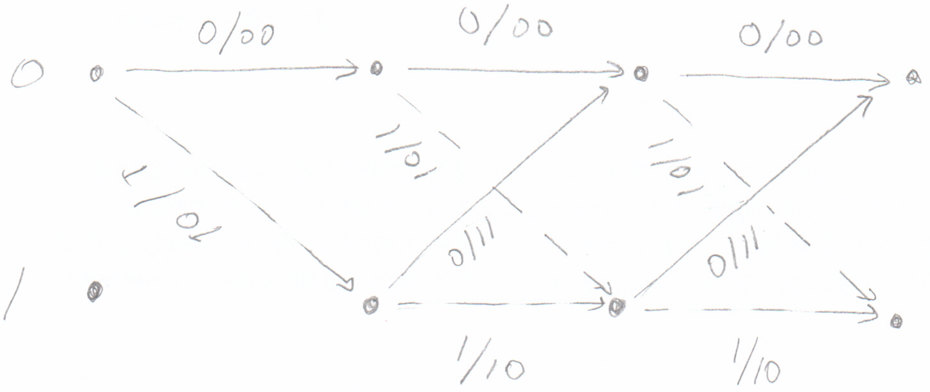
$g_1 = [0 \quad 1]$, $g_2 = [1 \quad 1]$

State Transition Diagram



Note that OP g_1 is scanned first

Trellis Diagram



If 110101 is fed as message sequence IP we find the OP a) By tracing, b) By interlacing c) From state Transition Diagram, d) From Trellis Diagram

Here we prefer b)

$$\text{Message sequence } X(p) = \overset{1}{p^5} + \overset{1}{p^4} + \overset{0}{0 \times p^3} + \overset{1}{p^2} + \overset{0}{0 \times p} + \overset{1}{1}$$

$g_1(p) = 1$, $X(p) g_1(p) = p^5 + p^4 + p^2 + 1$ -- xg_1

$g_2(p) = p+1$, $X(p) g_2(p) = p^6 + p^4 + p^3 + p^2 + p + 1$ -- xg_2

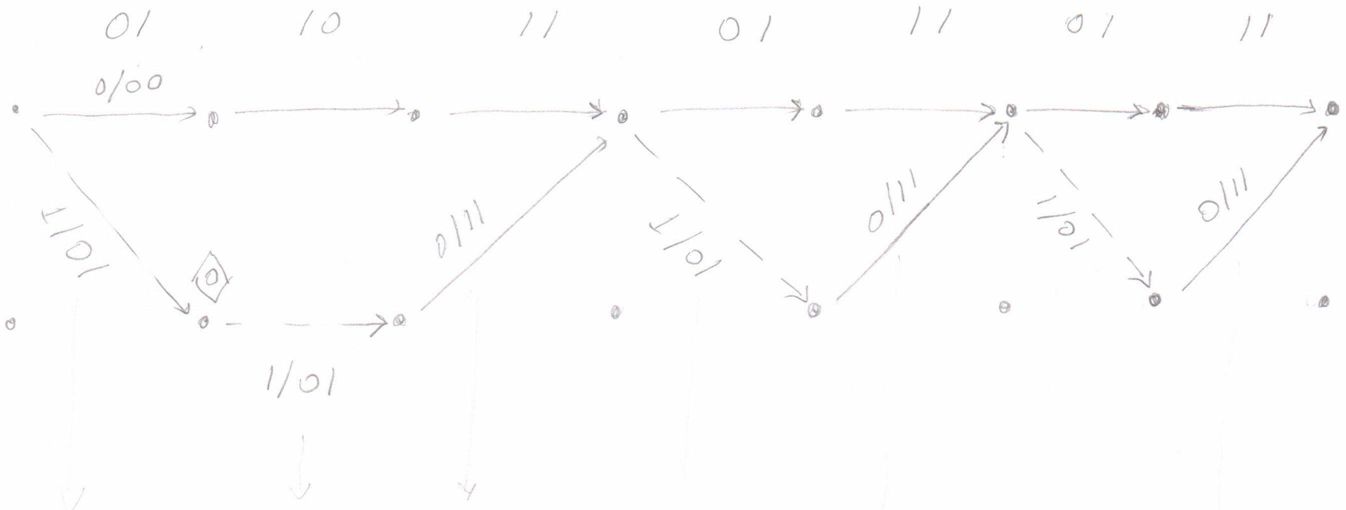
Interlace to get the encoded OP as

$$= \underbrace{\overset{xg_1}{0}\ \overset{xg_2}{1}}_{p^6}\ \underbrace{\overset{xg_1}{1}\ \overset{xg_2}{0}}_{p^5}\ \underbrace{1\ 1}_{p^4}\ \underbrace{0\ 1}_{p^3}\ \underbrace{1\ 1}_{p^2}$$

$$\underbrace{0\ 1}_{p}\ \underbrace{1\ 1}_{p^0}$$

ECE 587 - MT dated 7.04.2010 - Q1 (continued)

If this encoded sequence is fed to Trellis diagram decoding is acheived as follows



1 1 0 1 0 1 0

Original message sequence

Used to flush the register to zero state

Note that interlacing already takes into account the zero flashing produced upon decoding

2. (35 Points) Design a cyclic code of (6, 3). Find all the codewords, the Hamming distance, the minimum distance and the weight of the codewords of this code. Show how you verify that the generated codewords have cyclic property.

Solution: Generator polynomial has to be a degree of 3 and has to divide p^6+1 (without remainder)

$$p^6+1 = (p+1)(p+1)(p^2+p+1)(p^2+p+1)$$

$$= (p^3+1)(p+1)(p^2+p+1) = (p^3+1)(p^3+1)$$

So take $g(p) = p^3+1$, Hence we can construct the following table

Message Sequence	$X(p)$	Gen Poly $g(p)$	$X(p)g(p)$	Codeword
000	0	p^3+1	0	000000 — c_1
001	1	p^3+1	p^3+1	001001 — c_2
010	p	p^3+1	p^4+p	010010 — c_3
011	$p+1$	p^3+1	p^4+p^3+p+1	011011 — c_4
100	p^2	p^3+1	p^5+p^2	100100 — c_5
101	p^2+1	p^3+1	$p^5+p^3+p^2+p$	101101 — c_6
110	p^2+p	p^3+1	$p^5+p^4+p^2+p$	110110 — c_7
111	p^2+p+1	p^3+1	$p^5+p^4+p^3+p^2+p$	111111 — c_8

Hamming distance between the codewords differ from 2 up to 6, for instance

$d(c_1, c_2) = 2$, $d(c_1, c_8) = 6$

Hence $d_{min} = 2$. The weight of each codeword is the weight of non zero components, thus

$w(c_1) = 0$ —— $w(c_8) = 6$

It is easy to show that each codeword satisfies the cyclic property, such that any number of cyclic shift of any codeword is simply another (or the same) codeword

For instance

$c_4 = 011011$, after (1) cyclic shift

$c_4^{(1)} = 110110 = c_7$

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Frequency hopping is a type of spread spectrum system : True

Spread spectrum systems → Frequency hopping (FH)
→ Direct sequence (DS)

b) The result of multiplying the generator matrix of a linear block code by its parity check matrix is zero : False, the correct version is

$G H^t = 0$

Generator matrix

Transpose of a parity check matrix

c) Coding reduces dimensions of signals : False

(more BW requirement)

Coding increases dimensions of a signal set which means the end points of signal vectors are moved further apart

d) Multiplying the message signal with a spreading sequence spreads the signal into an narrower frequency spectrum : False

This operation causes the message signal to spread into a wider Bandwidth

e) Convolutional encoders have memory : True, the present output of the encoder not only depent on the present input but also previous ones, thus convolutional encoder has memory.